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A cellular automata model for highway traffic

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Abstract. We present results relative to a simple cellular automata model without periodic boundary conditions for an highway with on-ramps. Simulations performed with this model reproduce experimental phenomena observed in traffic such as free flow, synchronized flow, congested flow, lane inversion, forward and backward propagating waves. On-ramps play the important role of nucleation points for the dynamic features of traffic.

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1 Introduction

A cellular automaton consists of a regular lattice with a discrete variable at each site. A set of rules specify the time and space evolution of the system, which is discrete in both variables [1]. These systems have attracted much interest in very recent years because even with simple rules cellular automata may show very complex evolution patterns. These rules are usually, but not necessarily, limited to first neighbours interactions: that is, the state of a cell is completely determined by its nearest neighbours cells. The so-called game of life [2] is probably the simplest and best known example of cellular automaton which exhibits the potentiality of these systems. It is now recognized that repeated applications of simple rules can lead to very complex behaviours that can emulate physical, social and biological systems [3].

A recent application of cellular automata is car traffic simulation [4–7], which is a problem of great economical and social relevance. As physicists, we found particularly interesting to observe and try to understand how the complex features of traffic can emerge from a simple set of fundamental laws. One possible approach is to look for a dynamic model in terms of differential equations [8,9]. These models were able to discover that at high car densities traffic becomes unstable and free flow of cars turns into jams. These non linear equations also exhibit chaotic structures.

A different starting point is that of cellular automata. This method, that is simpler to implement on computers, provides a simple physical picture of the system and can be easily modified to deal with different aspects of traffic [10,11]. For these reasons, traffic cellular automata models are getting more and more popular.

Many results published up to now refer to cellular automata models with periodic boundary conditions. That is, the road has neither beginning nor end, and cars exiting the road at one end re-enter it at the other one. The use of periodic boundary conditions is a well known technique in physics to avoid mathematical complications. It is a clever approximation whenever the influence of the boundaries on the system is negligible, but, in the case of traffic, there are important effects influencing the dynamic of cars due to boundary conditions. The situation may resemble that of the liquid-vapor phase transition of water. Above the transition temperature a density fluctuation in the liquid could nucleate a bubble of the new phase and the transition could start. But, since the liquid is held in some container, in an actual experience the nucleation of the new phase invariably occurs at the container imperfections. The same is for traffic jams. They form whenever car density exceeds a certain value, but are nucleated by imperfections of the road.

It is common knowledge that there are frequently long queues at the off and on-ramps of an highway and experimental data [12] confirm that traffic jams propagating backward in space originate near intersections with other highways. Therefore we developed a cellular automata model for a two lane highway leaving periodic boundaries

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conditions in favor of a road with a beginning, an end and a certain number of on-ramps[1]. In our opinion such a non-periodic model represents an improvement with respect to periodic models in the following respects: 1) it is easier to implement in a computer program: cars are created at the desired rate by the use of a random number generator and there are no complications due to cars re-entering the road or car number conservation; 2) in order to study traffic effects due to car density increasing or decreasing in time, periodic boundary conditions can be a serious complication. Consider for example an highway with high car density which empties in time because few cars enter the road while many more leave it at its end. This common situation is treated straightforwardly in a non periodic model, but cannot be dealt with so easily in the case of periodic boundary conditions where cars leaving the road re-enter it at its beginning; 3) every highway has a beginning and an end. It is surely a better model the one which tries to reproduce as closely as possible the phenomenon which models. There is just one case where a periodic boundary model with on and off-ramps reproduces the correct topology of a road: the ring-shaped roads which usually surround cites. They are anyway different from highways because are shorter and with ramps much closely spaced than in the case of highways.

In the following, the non periodic model which we created will be described. We tried to keep it as simple as possible. This puts into evidence that the majority of the features of traffic are not due to an hypothetic complex behaviour of the drivers but to the repeated application of very simple rules. In spite of its simplicity, the model exhibits most of the experimentally known features of traffic.

2 Highway model

The model simulate a two lane highway with on-ramps as a long and straight road without traffic lights and crossroads where cars can in principle run without obstacles. The evolution rules were derived from those used by Schreckenberg and coworkers [13] and by Nagel and coworkers [7], which proved to be useful to implement cellular automata simulations of a multilane highway without ramps and using periodic boundary conditions. In this kind of simulations, cars have an unbound braking ability in order to avoid car accidents.

Each lane of the highway, whose length can be varied from simulation to simulation, is divided into cells which can be either empty or occupied by a car. The road starts at cell 1, and cars move in the direction of increasing cell number. The model is asymmetric: one lane, named in the following lane 2 or left lane, is used for overtaking while the other, named lane 1 or right lane, is for normal cruise. Cars enter the road at its beginning and at on-ramps. There are no off-ramps except for the highway end, where cars leave the road. On-ramps are modeled as a single entrance cell in lane 1. These cells are put from 1 500 to 2 500 cells apart and their number can vary from simulation to simulation. Typical numbers for on-ramps are from 4 to 8. For all on-ramps, at each time tick (tt) the program checks if the cell is empty. If empty, a random number in the [0,1] interval is generated and if it is less than the threshold chosen for that simulation, a car is generated in that cell with a speed of 2 cell/tt. In some cases, described below, cars were generated also in lane 2 with the same procedure. At the road beginning, cars are generated both on lane 1 and lane 2 with the procedure described above, with starting speed 2 cell/tt. For the sake of simplicity, cars can have only two values of their maximum speed; slow cars can be thought of as trucks. In all simulations presented in this work the percentage of slow cars created in lane 1 is 0.4 and is 0.15 for those created in lane 2. However, many simulations were performed with different values and different ratios for the number of slow cars in the two lanes without any qualitative differences in the results. As obviously expected, a greater percentage of slow cars eases the formation of congested traffic but do not introduce new phenomena.

The program, at the beginning of each simulation asks as input parameters highway length, on-ramp number, starting car density for each lane, car generation probability, random deceleration probability, fraction of slow cars, maximum speed of vehicles ($maxv$) and simulation length in time ticks. Calling $fgap1$, $fgap2$ and $bgap1$, $bgap2$ the forward and backward gaps (number of empty cells) between cars for lane 1 and lane 2; v, $fv2$ and $pv1$ car speed, speed of the following car on lane 2 and speed of the preceding car on lane 1, at each time step the system updates as follows:

For every car in lane 1

1) compute gaps.
2) if $v < maxv$: $v \rightarrow v + 1$;
3) if $v > fgap1$ and there is room enough on lane 2: change lane; else: $v = fgap1$.

For every car in lane 2

4) compute gaps.
5) If $v < maxv$: then $v \rightarrow v + 1$;
6) if $fv2$ or $pv1 > v$: return into lane1 as soon as possible.

For every car in lane 1 and lane 2

7) $v \rightarrow v - 1$ with given probability;
8) compute gaps (to take into account those cars which changed lane as a result of the previous steps);
9) On lane 1: if $v > fgap1$: $v = fgap1$;
10) On lane 2: if $v > fgap2$: $v = fgap2$;
11) update car position: if cell is full: cell $\rightarrow$ cell $+ v$.

At each time step it is possible to know a number of local and average quantities. For each car, position, speed and gaps are recorded. For each cell are computed mean

[1] Very recently, we become aware of a preprint article by G. Dietrich, L. Santen, A. Schadschneider, J. Zittark submitted on the International Journal of Modern Physics. In this article a single-lane highway with periodic boundary conditions and ramps is simulated with the use of cellular automata. This simulation seems to confirm the role of ramps as nucleation center for jams and other effects, as described later on in this work.

car speed and density, averaging over the 100 cells before and after the given cell. From these quantities the flow is readily computed as density by mean speed. The highway neither have a constant car density nor a constant car flow, which is exactly what happens in a real road, where these quantities usually chance in the time-scale of hours.

In order to compare the results of a simulation with experimental data it is needed to turn cells into meters, time ticks into seconds, speed from cell/tt into m/s and so on for all other quantities. With tt = 1 s and cell length = 5 m, fast cars with a speed of 9 cell/tt and slow ones with 6 cell/tt have a speed which corresponds to 162 and 108 km/h. A choice of different values do not change qualitatively the results of our simulations.

The computer program used for these simulations was written in Fortran90.

3 Results

The global traffic behaviour can be visualized by several plots such as: flow *versus* density, speed *versus* flow, lane occupation *versus* density, speed distribution *versus* density. In Figures 1a–1d these plots as obtained in one of our simulations are shown. A comparison with experimental data known from literature [7,14,15], shows that our model is able to reproduce the observed features of traffic.

Among those plots, that of car flow as a function of car density is called fundamental diagram [16]. Apart from a small number of scattered points (also present in both experimental and simulated data [7,14,15]), points ideally align along a first curve with positive slope at low densities and a second one with negative slope at high densities. This means that at low density, a density increase also increments flow, while the opposite happens at high density, where the more car are in, the slower they travel in the road. In this simulation, data seems to be rather scattered, but this is only apparent. A population level contour plot superposed to the same figure, reveals that most of the flow (about 99% of the points) concentrate in three regions. The different kinds of flow occurring in these regions will be examined in the following.

The elongated region on the left of Figure 1a corresponds to free flow [12], which is a condition where fast cars can easily overtake slow ones, there are large gaps between cars and traffic flows easily. It is the region of the fundamental diagram with positive derivative. Free flow can be generated introducing a low initial density in the road, a low car creation rate or, finally, eliminating all the on-ramps. In Figure 2, it is reported the density ratio distribution for a simulation in which the road is mostly in free flow conditions. The density ratio distribution, which informs on the lanes occupation, reveals that in free flow appears the phenomenon of lane inversion, that is a greater occupation of the left lane with respect to the right lane. This effect is due to the presence of a fraction of slow cars in the road, generally moving on lane 1. As a consequence, fast cars move and stay most of the time on the left lane in order to overtake. In Figure 3 it is represented the space-time distribution of lane inversion areas of a simulation with a car density slowly increasing in time.

An important result emerging from the simulations is that in a highway with cars entering the road at its beginning alone and moving without any obstacle except for the presence of other cars, free flow is the system attractor, that is the long term flow condition. The explanation for this fact is simply that a single on-ramp at its beginning is not enough to fill a highway. Once inside the highway, cars can accelerate in an empty road and rapidly leave the entrance area, so that the next cars to enter are free to accelerate. In these conditions a traffic jam cannot be generated and heavy traffic cannot be sustained, as reported in Figure 4. In the case of initial conditions of high density, a front is created at the road end between free flow moving cars and congested flow moving cars. The front propagates backward turning the road to free flow. We do not have real data to compare with these simulations, but from our experience as car drivers, we noticed that immediately after the beginning of an highway or after a queue due to an accident, traffic is fluid and car density is low.

Starting from the origin of the fundamental diagram and moving towards increasing densities, when the density is above 0.1 car/cell, local fluctuations can easily grow in amplitude and the system leaves free flow and usually enters a new phase of traffic, called synchronized flow. This phase was discovered by Kerner and Rehborn [12] and named synchronized flow by them. It is defined as a state of traffic in multilane roads in which the vehicles in different lanes move with almost the same speed. This speed is low with respect to that of cars in free flow (about 1/2), but higher than that of cars in heavy traffic, where it is close to zero. In this region flow can be high in spite of an increasing density, but the linear relationship between flow and density is lost and the two quantities becomes totally non-correlated. In the fundamental diagram, synchronized flow is mostly found in the contoured circular region at the centre of Figure 1a, even if it can also occurs at higher densities [12].

In Figure 5a, the space-time distribution of the above mentioned region of synchronized flow of Figure 1a is shown. The regions evidenced in the plot are those where cars have a speed v: $2 < v < 5$, a speed ratio r between the two lanes in the range $0.95 < r < 1.05$ and flow is between 0.45 and 0.55 car/(tt lane). In the simulation, regions of synchronized flow fades away in time (traffic turns into free flow in the left part of the road and congested flow in the right part) and are clearly triggered by the on-ramps, which are 7, 2 500 cell apart. This effect of the on-ramps is common to most dynamic features of traffic as simulated in our model and will be found again describing the nucleation and propagation of density and flow waves. Figure 5b shows the space-time distribution of synchronized flow regions obtained from a simulation with similar parameters but a lower initial density. Car speed, density ratio and flow are as for Figure 5a. Noteworthy, the synchronized flow regions seem to propagate both upward and downward in space, as reported for real traffic [12]. This phase

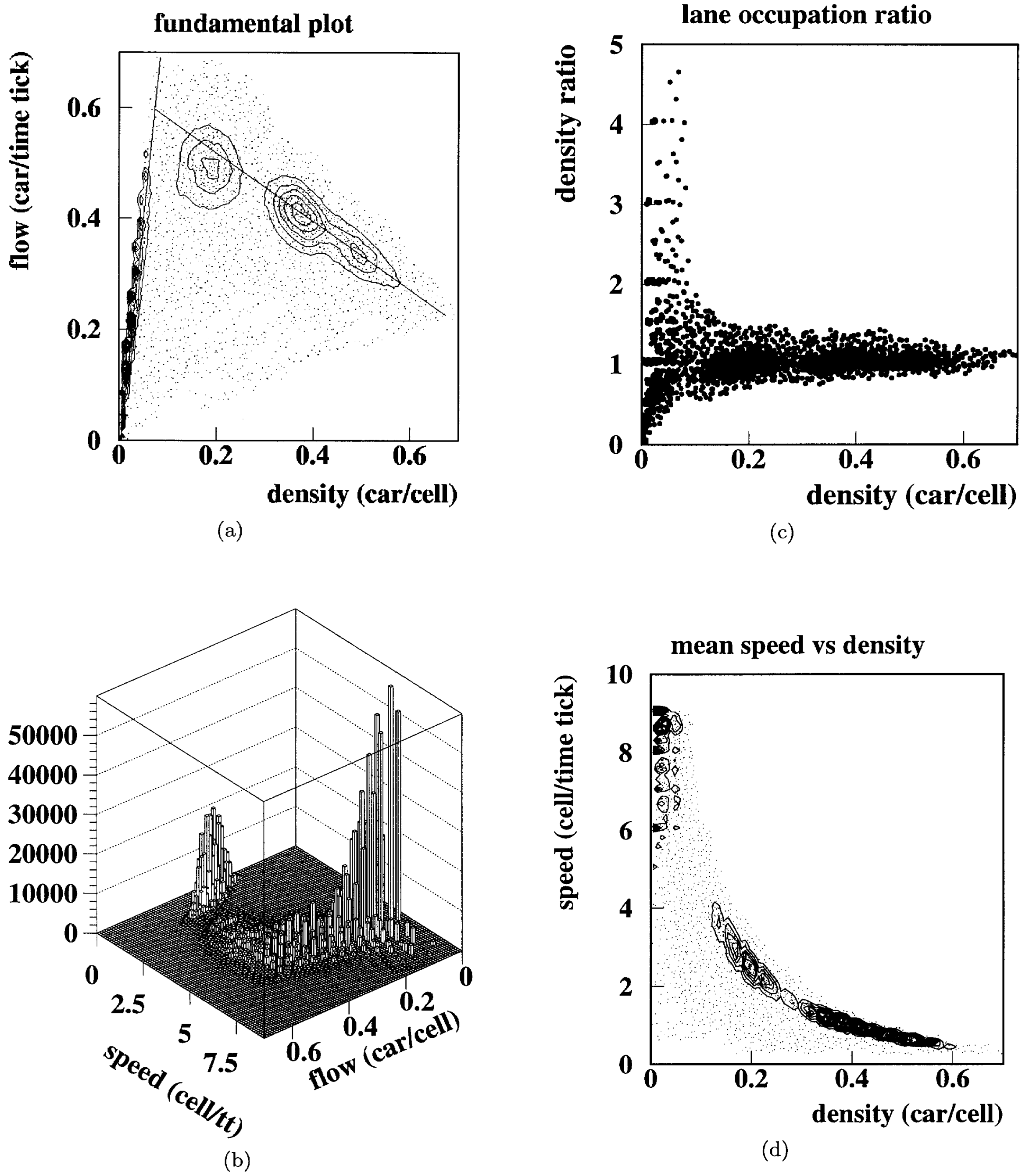


Fig. 1. (a) Car flow *vs.* car density. The population contour plot superposed to the data reveals how most points concentrate in three regions. Two lines are added to evidence free flow (line with positive slope) and congested flow (negative slope). They must be considered a simple guide for the eye. (b) Car speed *vs.* flow. In this figure, z axis instead of a contour plot is used to put into evidence the distribution of points over the bent curve. (c) Ratio between cars in lane 2 and in lane 1 *vs.* density of lane 2. At low densities it is often found a greater car occupation in lane 2 than in lane 1. (d) Mean speed *vs.* density in lane 2. At very low densities, cars are spread over the speed range between 6 and 9 cell/tt while at higher densities speed decreases as a function of density.

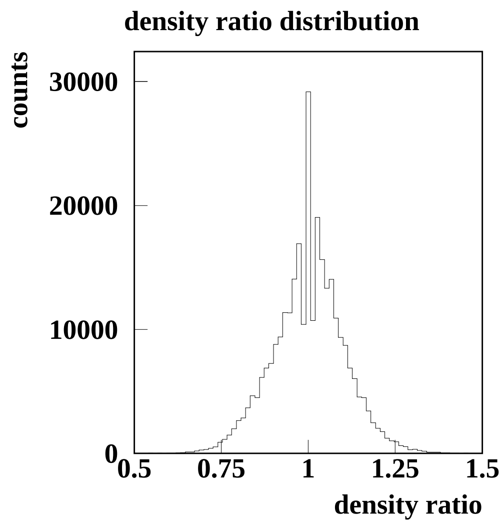


Fig. 2. Density ratio distribution between the two lanes.

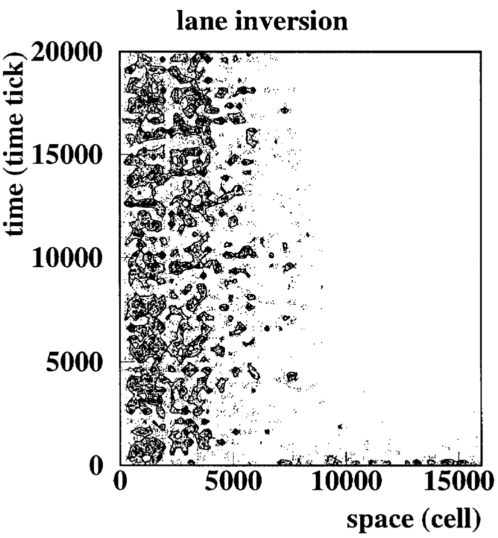


Fig. 3. Regions of space-time were lane inversion in free flow is observed $\left[(\text{density of lane2})/(\text{density of lane 1})\right] > 1)$. This effect is observed in the first part of the highway alone because, as a result of cars entering the road at the on-ramps, free flow disappears moving from left to right in space. In this simulation, the highway has 7 on-ramps 2 000 cell apart. A population contour plot is superposed to the data to better show the lane inversion areas.

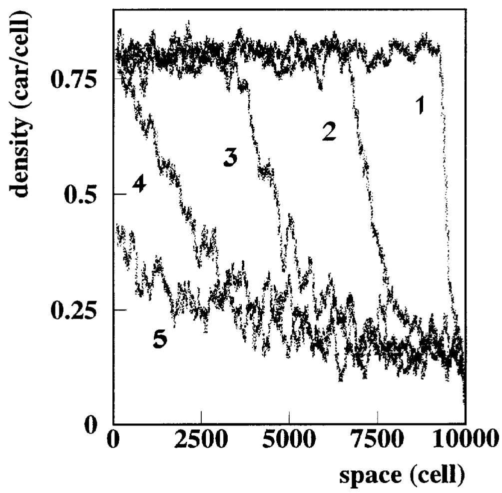
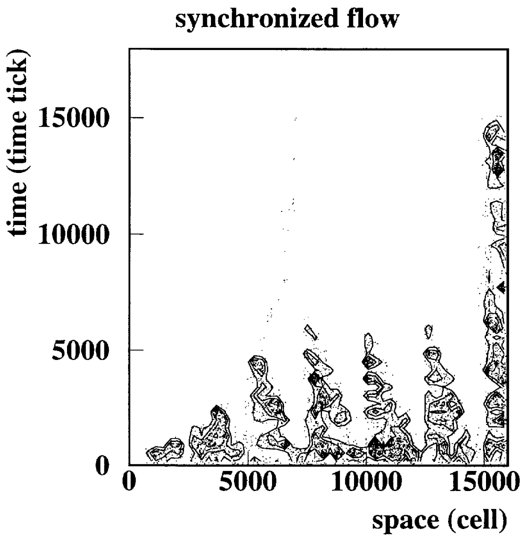


Fig. 4. Density *vs.* space at different times. In a highway with a starting density of about 0.8 car/cell and with only one on-ramp at the road entrance, density decreases with time. The five curves reported in figure refer to density in lane 1 after 1 000 (1), 5 000 (2), 10 000 (3), 15 000 (4), 20 000 (5) time ticks. Propagation speed is the same as that of backward propagating waves.

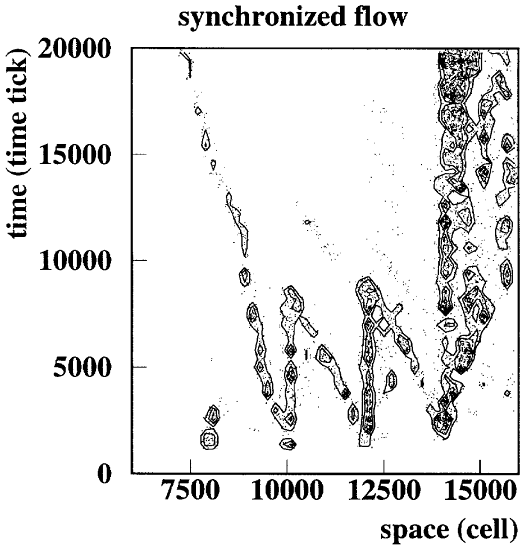
of traffic occurs, in our simulations, at speeds lower than the maximum speed of slow cars and is not due to the presence of 2 kinds of cars.

At higher densities ($\rho > 0.3$ car/cell) we enter the congested flow region of the fundamental diagram, see Figure 1a, where flow is getting lower and lower as the density increases (negative slope). Traffic jams, that is regions where cars slow down even to a complete stop and the density grows nearly to its maximum, easily form. Jams propagate backward like waves with a simple mechanism: at low speed a car or group of cars may stop; because of the finite time required for a stopped car to start moving again and the closeness of cars in the road, the following cars join the jam, that is are forced to stop, while cars at the opposite end of the jam start moving, leaving the jam. In this way, cars join and leave the jam downstream, and the region of bundled cars moves upstream. Figures 6, 7 and 8 show jams produced in our simulations and propagating at a speed of $-(11 \div 12)$ km/h, assuming a cell length of 5 m. The same range of speeds have been measured in the fundamental plots of Figures 1a and 7c for the slope of the flow *vs.* density curve in the congested flow region. As evidenced by the population contour plot of Figure 7c, most cars of that simulation travel in congested flow. In such a flow, car distribution is narrower than that reported in Figure 2 because of the absence of lane inversion, see Figure 7d.

As stated before, jams form near the on-ramps and afterward remain stable in their propagation through all the on-ramps [18]. In the simulation used to produce Figure 6, there are 5 on-ramps 2 500 cell apart. The simulation of Figures 7 and 8 has only 2 on-ramps 10 000 cell apart, with cars entering the road at both lanes. In practice parameters are chosen in this simulation so that the second on-ramp acts as a gateway. A jam produces at the gateway and moves backward in space up to the highway beginning. This wave has a non dispersive character, like a soliton, and this confirms the non linear character of the flow equations underlining our model. Solitons are special solutions of the Korteveg-De Vries equation [19] but there are many other non linear differential equations that admit single wave solutions that are not solitons. Further work is in progress to better understand the character of the waves that we observe in our model.



(a)



(b)

Fig. 5. (a) Traffic as a function of space and time. In this plot, the regions of synchronized flow ($2 < \text{speed} < 5$ cell/tt, speed ratio r between the two lanes in the range $0.95 < r < 1.05$ and $0.45 < \text{flow} < 0.55$ car/(tt lane)) are reported with population contour lines added to help visualizing the areas of interest. (b) Synchronized flow in a simulation with a different number of on-ramps and slightly lower starting density. There is evidence of forward and backward propagating waves as well as of steady states near on-ramps.

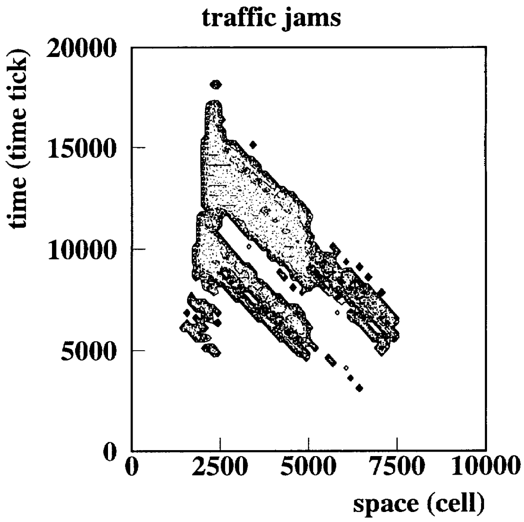


Fig. 6. Traffic as a function of space and time. In the plot are only reported regions where the density of lane 2 was greater than 0.75 car/cell and the flow was less than 0.2 car/(tt). In a situation of globally decreasing car density as a function of time, near the on-ramps at cell 5 000 and 7 500 a jam forms and subsequently propagates backward in space. This simulation was 18 000 tt long.

Despite the many simulations, performed with different parameters, jams always born at the highway on-ramps. This can be regarded as a proof of the statement that even if in principle jams can born spontaneously, in real traffic they are triggered by roadwork, junctions, accidents, ramps and similar imperfections of the road. The lack of off-ramps in our model is linked to this. In fact, off-ramps influence traffic mainly producing a local slowing down of car speed which, in heavy traffic, triggers jams. Another effect is, of course, to decrease the number of cars in the street. The first effect is already introduced in our model by the presence of on-ramps, while using different starting conditions (car generation and car density), our model is equally able to yield traffic with increasing or decreasing car density. We were even able to produce conditions were one half of the highway has increasing car density while the other half has decreasing car density. Therefore, to keep the model as simple as possible, off-ramps were avoided.

Jams do not exhaust all the feasible waves in traffic. There are both upstream and downstream moving waves in synchronized flow and downstream moving waves in free flow [12]. Our model was able to produce both kind of waves, as shown in Figure 5b and Figure 9. The mechanism which gives rise to waves in free flow is different from that of jams. Now the average car speed is rather high (> 5 cell/tt) and the overall density low (< 0.2 car/cell). Considering lane 2, fast cars reach and stuck after slower cars when for some reason the latter do not give way fast enough. This creates a moving region denser than the average in which cars join (reached by faster cars) and leave (left behind by the faster ones) the wave upstream while the wave moves downstream.

Apart from reproducing the global features of traffic and many dynamic structures, the simulations put into evidence the noisy structure of the variables used to describe traffic. Speed, density, flow: all of them, when plotted as a function of time (at fixed space) or space (at fixed time) exhibit a fragmented trend which seems to repeat at different scales. Work is in progress to check if the concept of fractal is appropriate for these quantities and to measure their eventual fractal dimension.

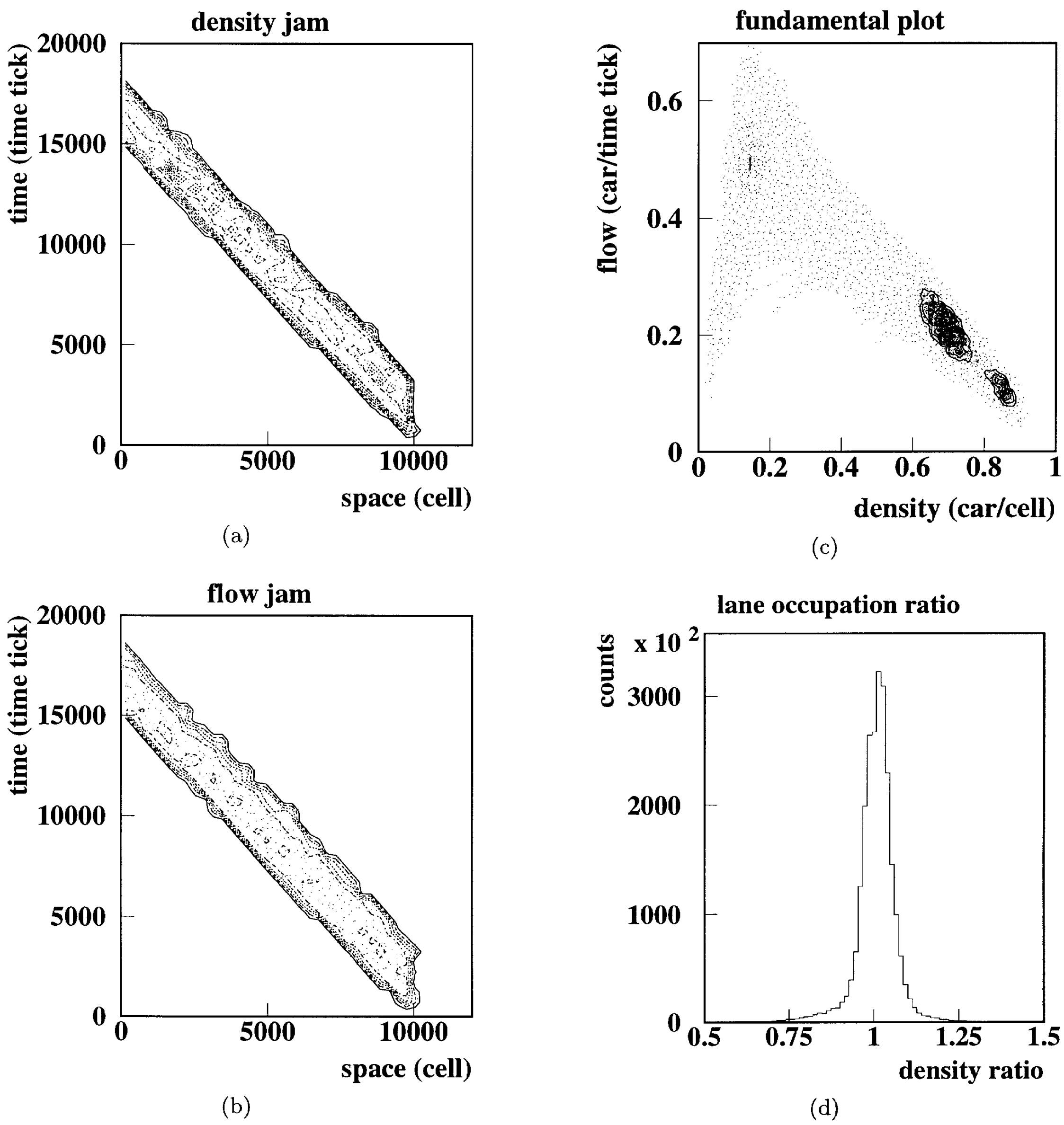


Fig. 7. (a) Traffic as a function of space and time. In the plot is only reported the region where the density of lane 2 is greater than 0.85 car/cell. In a highway with starting density of 0.65 car/cell in each lane, a traffic jam forms at the on-ramp at cell 10 000 and propagates backward along the highway. (b) The same jam observed as a flow wave. In the plot the region with flow < 0.15 car/(tt) is reported. (c) Fundamental plot relative to the same simulation. The population contour plot reveals that most cars are found in the area centered about a density value of 0.7 car/cell. The smaller region above 0.8 car/cell is due to cars in the jam. (d) Density ratio distribution between the two lanes. In congested flow the two lanes are equally occupied and the distribution is narrower than that of Figure 2.

4 Conclusions

With the use of this simple cellular automata model of a highway, we were able to reproduce both global features of traffic, such as fundamental plot, and dynamic features of traffic such as the creation and propagation of waves. In this respect, we would like to emphasize the simulation of forward propagating density waves in free flow (see Fig. 9) and that of non dispersive waves in congested flow (see Fig. 7). The speed of the backward propagating waves in congested flow can be obtained from the slope of the flow versus density plot (see the line with negative slope of Fig. 1a).

The other important point of this research is the simulation of a realistic highway, without periodic boundary conditions and with a variable number of on-ramps. We believe that a model with periodic boundary conditions and ramps could reproduce some effects such as synchronized flow or waves in free flow, which are usually not observed in models with periodic boundary conditions and

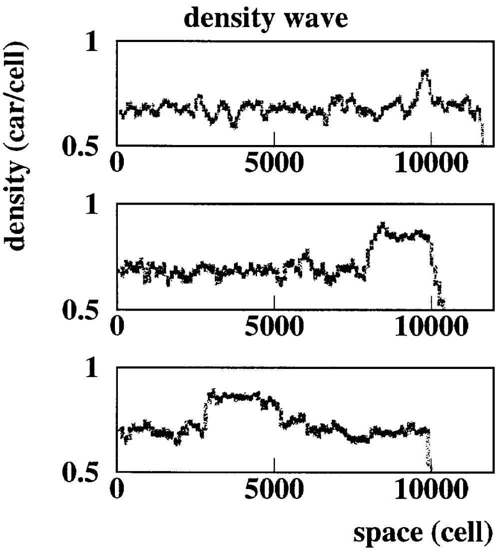


Fig. 8. Growth and propagation of the wave described in the previous figure. From top to bottom density of lane 2 as a function of space at time 600, 3 000, 11 000 tt.

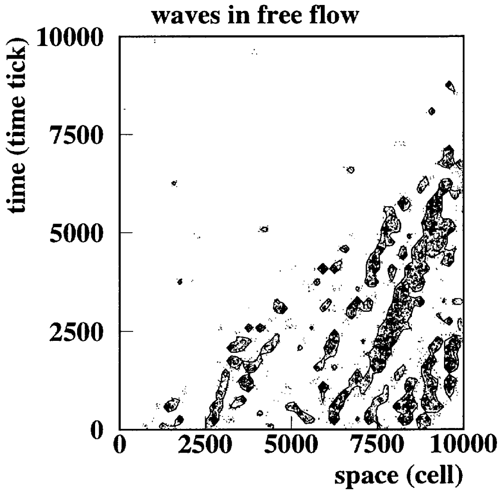


Fig. 9. Waves in free flow in a simulation with only one on-ramp at cell 1. In this plot the regions of space-time where (density of lane 2) > 0.1 car/cell are reported. The starting density of the two lanes is 0.1 car/cell. Regions denser than the average form and propagate forward along the highway only at low tt because of globally decreasing car density.

no ramps, but we are decisely in favor of a non periodic model because it is closer to real highways and because in such a model it is straightforward to simulate traffic conditions with a variable number of cars travelling the road. In other words, in models like the one we used the dynamic or transient features of traffic are better reproduced, as demonstrated by the results which we presented in this article. In particular, we were able to show that car entering the road from on- ramps is required to obtain congested flow and that jam waves always form at ramps and then propagates along the road. It is interesting to note that forward propagating waves in free flow seem, on the contrary, to form spontaneously.

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